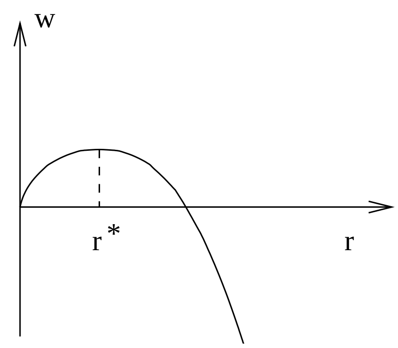
Question B1 The driving force for nucleation ΔG_v is controlled by the temperature of annealing. It is low close to the transition temperature, high at lower temperatures where the transformation might however be inhibited by low diffusion rates. Producing a nuclei 'costs': interfacial energy between crystal and glass + elastic strain energy (low if glass is of same composition).

For a nuclei to be stable, its growth must be accompanied by a decrease in Gibbs free energy. Consider the work done to create a nuclei of radius r:

$$W = 4\pi r^2 \sigma + \frac{4}{3}\pi r^3 \Delta G_v$$



where ΔG_v is negative if the transformation is to occur. The critical radius is given by $\frac{\partial W}{\partial r} = 0$, ie:

$$r^* = -\frac{2\sigma}{\Delta G_v}$$

What is the critical radius ? It is the radius above which a nuclei is stable and therefore able to grow. Any nuclei smaller than r^* will disappear. The critical radius depends on the temperature: the lower the temperature (with regard to the transformation one), the larger the driving force, therefore the smaller r^*. We can clarify the temperature dependency of ΔG_v:

At equilibrium:

$$\Delta G_v(T_{eq}) = 0 = \Delta H_v(T_{eq}) - T_{eq}\Delta S_v(Teq)$$
$$\Rightarrow \Delta H_v(T_{eq}) = T_{eq}\Delta S_v(Teq)$$
$$\Rightarrow \Delta G_v(T) = \Delta H_v(T_{eq})\left(1 - \frac{T}{T_{eq}}\right)$$

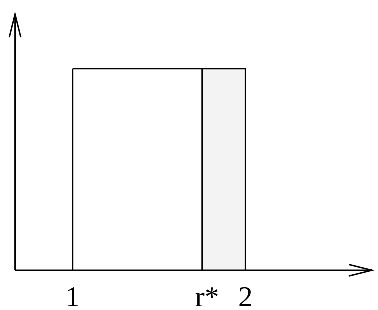
where in fact we assume that ΔH_v and ΔS_v are not changing with temperature. This is reasonable within a limited range of temperature.

Obtaining a grain size of $40\mu m$: this is a simple three step problem: (1) find the number of nuclei we need to grow to produce this grain size (2) express it as a percentage of the total number of nuclei formed and find the critical radius we want, (3) once we know what we want r^* to be, we calculate the temperature that will give it.

(1) We need to assume that a unit volume is filled up by cubic grains, each grain comes from one nuclei. These grains can be taken to have a side length of $40\mu m$, therefore each occupy a volume of $(4.10^{-5})^3$ and the number of nuclei (per unit volume) is the number of times we can fit this volume into a unit volume.

$$1/(4\,10^{-5})^3 = 0.1562\,10^{14}$$

(2) The graph shows that the nuclei are uniformly distributed by size between 1 and 2 nm, the total number of nuclei per unit volume being 10^{14}. We need to find r^* which will lead to the growth of $0.1562\,10^{14}$ nuclei. Nuclei which grow are located at radii larger than r^* as illustrated below:



That means the ratio $\frac{2-r^*}{(2-1)}$ should be equal to $\frac{0.1562\,10^{14}}{10^{14}}$, which gives:

$$r^* = 1.84nm$$

(3) Using the expression for r^*, obtain the wanted driving force. Using the expression for the change of ΔG_v with temperature, calculate the temperature giving the wanted driving force. We obtain:

$$\mathrm{T} = 1161^\circ C$$