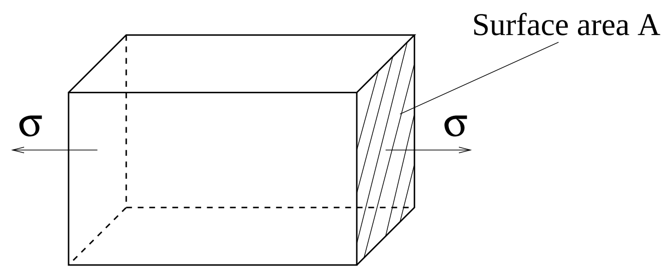
Question A4, 1996:

Stiffness: when considering an inter-atomic bond as a spring of energy $U(r)$, where r is the inter-atomic spacing, one can define the stiffness as:

$$S = \frac{\mathrm{d}^2U}{\mathrm{d}r^2}|_{r=r_0} = \frac{\mathrm{d}F}{\mathrm{d}r}|_{r=r_0}$$

In the region of elastic deformation, isotropic materials exhibit a linear relationship between the stress σ and the deformation ε, written $\sigma = E\varepsilon$, where E is the Young's modulus of the material.

Given:



for an elastic deformation:

$$\sigma = E\varepsilon$$

where $\varepsilon = \frac{\delta l}{l_0}$. This is also giving the increase in inter-atomic distance for a bond parallel to the tensile axis: $\delta r = \varepsilon r_0$. The force on such a bond is therefore:

$$F = S\delta r = S\varepsilon r_0$$

Over the area A, there are A/r_0^2 bonds, so that the overall force is:

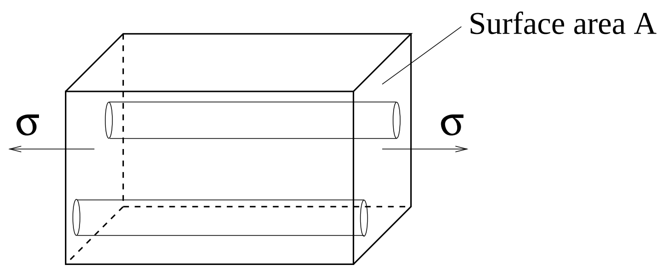
$$F\frac{A}{r_0^2} = \sigma A$$

by replacing σ and F, we obtain:

$$E = \frac{S}{r_0}$$

The difference in Young's modulus for different materials containing only C-C bonds can be explained by the nature of the bonds and the orientation with regard to the tensile axis: carbon only contains covalent bonds, carbon fibre are made up of graphite but oriented so that the weak interplanar bonds are perpendicular to the stress, this is not the case of conventional graphite for which the value is an average over the strong 2d covalent bonds and the weak interplanar bonds. For PE, there is only one direction along which C-C bonds are of covalent nature, other bonds are VdW interactions.

Apparent Young's modulus of a composite:



We wish to identify an apparent Young's module defined by: $\sigma_0 = E_{app}\varepsilon$. Assuming that the interface between the PE (polyethene) and the CF (carbon fibres) is perfect, the elongation ϵ is the same for both materials. The overall force F_0 corresponding to the stress σ_0 can therefore be decomposed as:

$$F_0 = \sigma_0 A = E_{pe}\varepsilon\ V_{pe}A + E_{cf}\varepsilon\ V_{cf}A$$

where V_{cf} is the volume fraction of fibres and $V_{pe} = 1 - Vcf$ that of polyethene, and noting that $V_{pe}A$ is the fraction of the surface A occupied by polyethene. Therefore the apparent Young's modulus is given by:

$$E_{app} = V_{cf}E_{cf} + (1 - V_{cf})E_{pe}$$